

MA114 Summer 2018
Worksheet 11 – Alternating Series – 6/26/18

1. (a) Let $a_n = \frac{n}{3n+1}$. Does $\{a_n\}$ converge? Does $\sum_{n=1}^{\infty} a_n$ converge?
(b) Give an example of a divergent sequence $\sum_{n=1}^{\infty} a_n$ where $\lim_{n \rightarrow \infty} a_n = 0$.
(c) Is there any example of a convergent sequence $\sum_{n=1}^{\infty} a_n$ where $\lim_{n \rightarrow \infty} a_n \neq 0$?
(d) Suppose we have an alternating series $\sum_{n=1}^{\infty} (-1)^{n+3} a_n$, where $a_n \geq 0$. Is it possible that the series diverges?
2. Decide whether the Alternating Series Test can be used to show that the following series converge. If it cannot, explain why and if possible use a different test to determine whether the series converges or not.

(a)
$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2n}$$

(b)
$$\sum_{k=2}^{\infty} (-1)^{k+1} \frac{1}{\ln(k)}$$

(c)
$$\sum_{m=2}^{\infty} \frac{3^m}{4^m + 5^m}$$

(d)
$$\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln(n)}$$

(e)
$$\sum_{n=1}^{\infty} (-1)^n \frac{\cos^2(n)}{n}$$

(f)
$$\sum_{i=1}^{\infty} \left(\frac{-5}{18} \right)^i.$$

3. Estimate the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n}$ correct to three decimal places, i.e. so that the absolute error is at most 0.0005.